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Three-Dimensional Rigid-Plastic FEM Simulation of Metal Forming Processes

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A three-dimensional finite element modeling (FEM) code for simulation of bulk forming processes has been developed. Some techniques, such as die surface description, contact judgment algorithm, and remeshing, are proposed to improve the robustness of the numerical solution. To verify the proposed method, the isothermal forging process of a cylindrical housing has been simulated. The simulation results show that the given techniques and FEM code are reasonable and feasible for three-dimensional bulk forming processes.

Keywords B-spline, contact judgment algorithm, finite element modeling (FEM), metal-forming process, remeshing

1. Introduction

During the metal-forming processes, most three-dimensional parts involve complicated geometry and deformation characteristics. A schematic study for the forming processes is needed to improve the product quality, decrease the cost, and shorten the development time. Nowadays, numerical simulation using the three-dimensional rigid-plastic finite-element method (FEM) introduced by Kobayashi and Lee (Ref 1) in the early 1970s has become a powerful and economical tool to analyze various kinds of forming processes. It has been widely used for process design, defect prediction, and optimization (Ref 2-5). However, most three-dimensional forging components involve complicated geometry and deformation characteristics in the bulk forming processes. It is difficult to simulate the forming processes automatically and systematically. Especially when a simple billet is forged into a complicated final component, simulation requires some intermediate remeshing procedures due to rapid mesh degeneracy. At present, some remeshing algorithms have been proposed for simulation of three-dimensional forming processes (Ref 6-10). However, requirements related to proper computational economy and physical characteristics are yet not satisfied. In addition, a lot of time is needed to define the complex boundary conditions, including the description of die surface, the contact judgment between nodal points and die surface, and the imposition of friction. Improper boundary conditions increase the instability of numerical simulation.

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In this paper, a new remeshing method that can generate eight-node hexahedral element is proposed. Some key schemes such as die description and contact judgment are discussed also.

2. Rigid-Plastic FEM Formulation

Rigid-plastic material model is commonly used to simulate bulk metal forming processes. Although it is not possible to calculate residual stresses and spring-back, the outstanding advantage of the material model is the stability and low cost of the numerical solution scheme.

The basic equations of the rigid-plastic finite element are as follows:

Equilibrium equation:

$$\sigma_{ij,j} = 0 \qquad \text{(Eq 1)}$$

Compatibility and incompressibility condition:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad \dot{\varepsilon}_v = \dot{\varepsilon}_{ii} = u_{i,i} = 0 \qquad \text{(Eq. 2)}$$

Constitutive equation for the rigid-plastic material:

$$\sigma'_{ij} = \frac{2}{3}\frac{\bar{\sigma}}{\dot{\bar{\varepsilon}}}\dot{\varepsilon}_{ij}, \qquad \bar{\sigma} = \sqrt{\frac{3}{2}(\sigma'_{ij}\sigma'_{ij})}, \qquad \bar{\varepsilon} = \sqrt{\frac{2}{3}(\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})} \qquad \text{(Eq 3)}$$

Boundary conditions:

$$\sigma_{ij} n_i = F_j \quad \text{on} \quad S_F, \qquad u_i = U_i \quad \text{on} \quad S_v \qquad \text{(Eq 4)}$$

where σ_{ij} and $\dot{\varepsilon}_{ij}$ are the stress and strain rate components, respectively. $\bar{\sigma}$ and $\dot{\bar{\varepsilon}}$ are the equivalent stress and the equivalent strain rate.

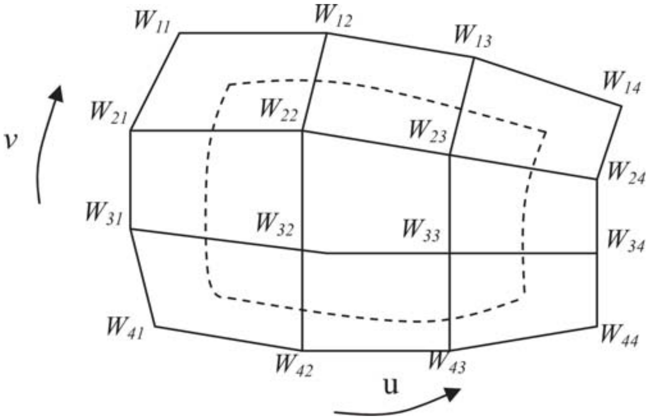

Fig. 1 Piece of cubic B-spline patch and its control grid

From the variational principle the functional of the rigid-plastic material model can be written as follows:

$$\Phi = \int_V \bar{\sigma}\dot{\bar{\varepsilon}} dV - \int_{S_F} F_i u_i dS + K \int_V \frac{1}{2} \dot{\varepsilon}_V^{\,2} dV \qquad \text{(Eq 5)}$$

where F_i denotes the force of the given surface on the force surface S_F, U_i denotes the deformation velocity on the boundary surface S_v. K is a penalty constant to neglect the incompressibility condition, and $\dot{\varepsilon}_V$ is the volume strain velocity, $\dot{\varepsilon}_V = \dot{\varepsilon}_{ii}$.

The friction boundary condition is given by:

$$\mathbf{f} = -\frac{2}{\pi} mk \cdot \tan^{-1} \frac{|u_s|}{u_0} \cdot \mathbf{t} \qquad \text{(Eq 6)}$$

where m is the friction factor, k is the yield shear stress, u_s is the sliding velocity of the work piece relative to the die, u_0 is a small positive number compared with u_s, and t is the unit vector in the opposite direction of the relative sliding velocity.

The variational functional (Ref 5) can be converted to a set of nonlinear algebraic equations that can be solved by the Newton-Raphson method.

3. Die Description and Contact Algorithm

3.1 Die Description

In many bulk-forming processes, die surface is so complex that it cannot be described with analytic expression. Considering the flexibility and universal suitability of B-spline, in this paper, this method is adopted to describe the die surface. First, the die surface is divided into several surface patches and every patch is described by double cubic even B-spline. The goal of double cubic even B-spline is to obtain a smooth die surface, both within a patch and at its boundaries. Finally, the surface patches are divided into triangular elements and the normal vector of each element is provided at the same time. This description can present convenient and flexible numerical geometrical information to process dynamic boundary conditions in a finite element method.

In this kind of description, as shown in Fig. 1, the surface patch can be described by using a control grid matrix **W**. Each patch can be defined as a function of two parameters, u and v, written as:

$$\mathbf{P}(u, v) = \sum_{i=0}^{3} \sum_{j=0}^{3} N_{i,3}(u) N_{i,3}(v) W_{i,j} = \mathbf{UBW}^{\mathrm{T}}\mathbf{BV}^{\mathrm{T}}$$
$$(0 \le u, v \le 1,\ i, j = 0, 1, 2, 3) \qquad \text{(Eq 7)}$$

where $N_{i,3}$ is the B-spline basic function, $W_{i,j}$ are the elements of a control grid matrix **W**, and:

$$\mathbf{U} = [1 \quad u \quad u^2 \quad u^3], \quad \mathbf{V} = [v_i \quad v_{i+1} \quad v_{i+2} \quad v_{i+3}],$$

$$\mathbf{B} = \frac{1}{6}\begin{bmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \qquad \text{(Eq 8)}$$

3.2 Contact Algorithm

In three-dimensional deformation problems, the contact between the nodes and the die surface must be checked carefully to obtain accurate solutions. A method is presented to judge the contact between an arbitrary node and an arbitrary die triangular element. First, four vectors should be defined, **N** is the normal vector of the die's triangular element whose node code is anticlockwise, **V** is the velocity vector of deforming node Q, **QE** is a vector pointing from die's triangular element plane to node Q, **PF** is a vector pointing from die's triangular element plane to node P, which is the new position of node Q after the iteration step. As shown in Fig. 2, there occur six instances among the four vectors.

(1) $\mathbf{N} \cdot \mathbf{QE} > 0$ and $\mathbf{N} \cdot \mathbf{V} > 0$, node Q deviates from the element.
(2) $\mathbf{N} \cdot \mathbf{QE} > 0$ and $\mathbf{N} \cdot \mathbf{V} < 0$, node Q moves toward the element. There occur two instances, if $\mathbf{N} \cdot \mathbf{PF} > 0$, the node could not contact the element, if $\mathbf{N} \cdot \mathbf{PF} < 0$, the node might contact the element.
(3) $\mathbf{N} \cdot \mathbf{QE} < 0$ and $\mathbf{N} \cdot \mathbf{V} < 0$, node Q deviates from the element.
(4) $\mathbf{N} \cdot \mathbf{QE} < 0$ and $\mathbf{N} \cdot \mathbf{V} > 0$, node Q moves toward the element. There occur two instances, if $\mathbf{N} \cdot \mathbf{PF} < 0$, the node could not contact the element, if $\mathbf{N} \cdot \mathbf{PF} > 0$, the node might contact the element.

In fact, only when $\mathbf{N} \cdot \mathbf{QE} > 0$, $\mathbf{N} \cdot \mathbf{V} < 0$, $\mathbf{N} \cdot \mathbf{PF} < 0$ or $\mathbf{N} \cdot \mathbf{QE} < 0$, $\mathbf{N} \cdot \mathbf{V} > 0$, and $\mathbf{N} \cdot \mathbf{PF} > 0$ could the node Q contact the die's element. After this search, all die's elements who might contact the node Q can be selected, then a local search is performed to select the element that will contact with node Q.

Node Q and P are projected on the selected die's triangular elements and the crosspoint K of line QP and die's triangular element plane is calculated. The area difference of triangles is used to accurately judge the correlation of point E, F, K, and die's triangular element. If points E, F, and K are within the triangle element or F and K are within the triangle element, node Q will contact with the die's triangular element after this iteration step and the point F is a new position of node Q. Otherwise, if point K is within the triangle element, node Q

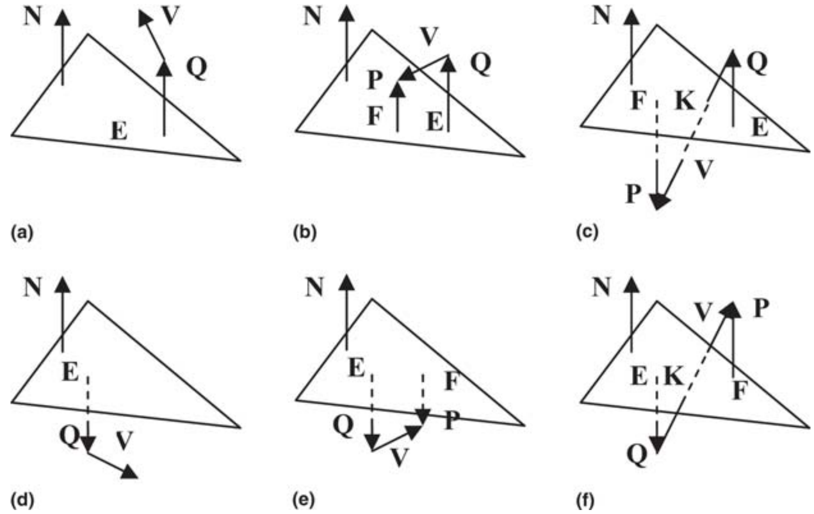

Fig. 2 Relationship between node and the die's triangular element: (a) **N** · **QE** > 0, **N** · **V** > 0; (b) **N** · **QE** > 0, **N** · **V** < 0, **N** · **PF** > 0; (c) **N** · **QE** > 0, **N** · **V** < 0, **N** · **PF** < 0; (d) **N** · **QE** < 0, **N** · **V** < 0; (e) **N** · **QE** < 0, **N** · **V** > 0, **N** · **PF** < 0; (f) **N** · **QE** < 0, **N** · **V** > 0, **N** · **PF** > 0

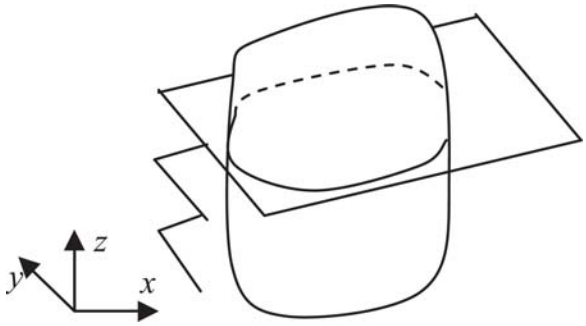

Fig. 3 Generation of cross sections in the volume

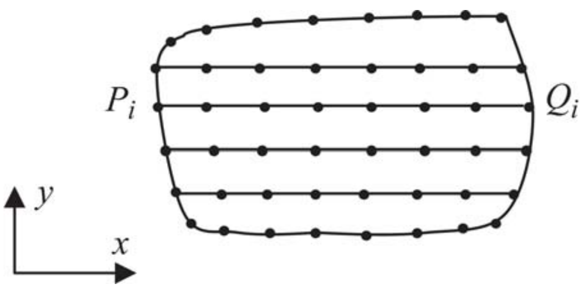

Fig. 4 Generation of nodes on the cross sections

Fig. 5 Photograph of cylindrical housing

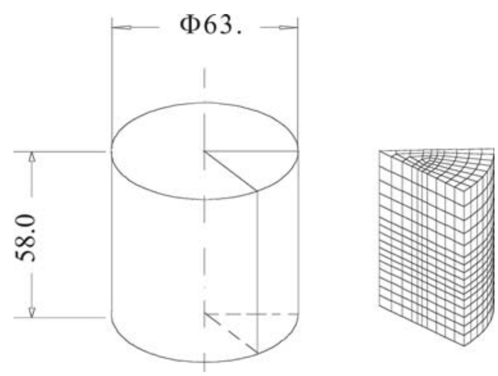

Fig. 6 Initial billet and its mesh system

will contact with the die's triangular element and the point K is a new position of node Q.

4. Remeshing

In this paper, a simple but versatile remeshing scheme is proposed. The volume Ω, which is made up of the eight-node hexahedral elements, is supposed to be rezoned. Planar cross sections can be obtained when a series of parallel planes cuts across the object. Nodes of given volume Ω are generated on these cross sections using the same method as for the generation of nodes on two-dimensional planar domains. Finally, three-dimensional eight-node hexahedral elements can be constructed by connecting the nodes on two adjacent cross sections.

To define the outside surfaces of the volume, the boundary surface of the boundary element is divided into two triangular facets for which the normal vectors, as defined by the rigid-hand rule, point outward. Therefore, the boundary surfaces of volume Ω can be defined solely by a collection of triangular elements. The following section describes the main procedures of this scheme.

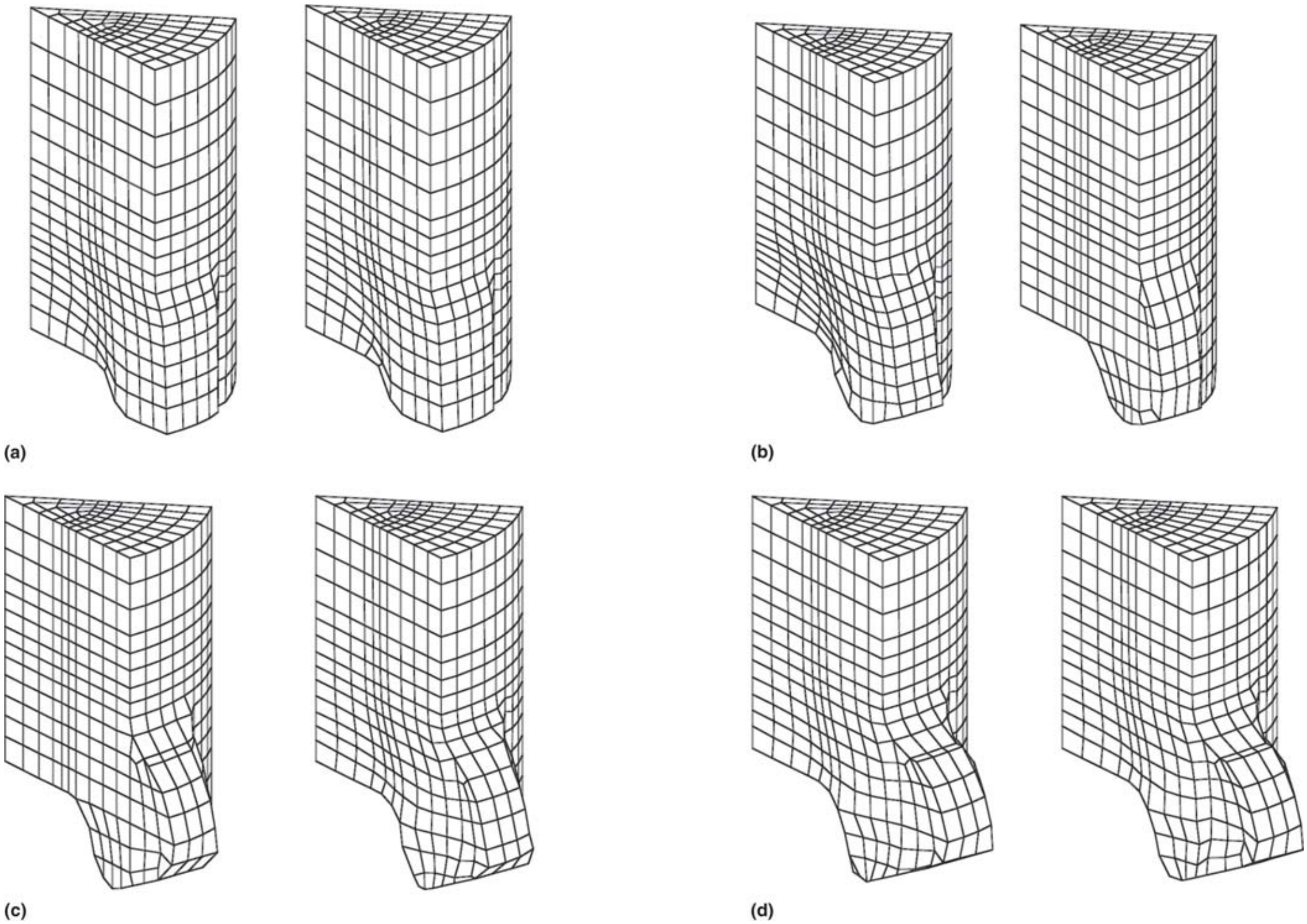

Fig. 7 Mesh configuration before and after remeshing at (a) 10.3% reduction, (b) 18.6% reduction, (c) 25.9% reduction, and (d) 32.3% reduction

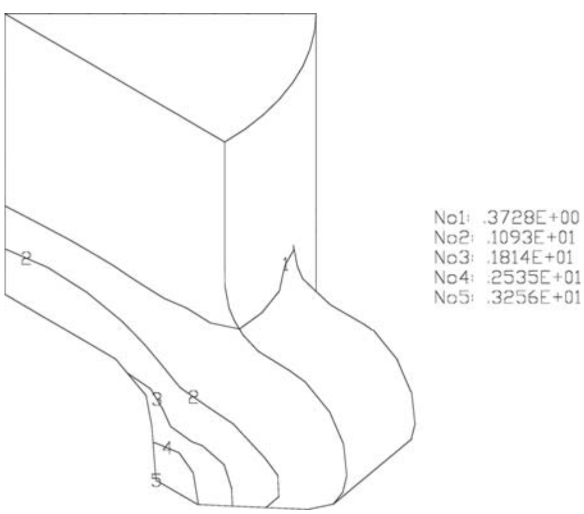

Fig. 8 Equivalent strain distribution at the final stage

4.1 Dividing Subregions

During the three-dimensional forming processes, billet undergoes large volume transfer, and intermediate shape is usually complicated. To deal with arbitrary domain easily, the volume is first divided into several subregions according to its geometry and flow characteristics, and then remeshing is done on every subregion respectively.

4.2 Generating Nodes

Assuming that a series of parallel planes are used to cut volume Ω, several cross sections can be obtained, as shown in Fig. 3. The assuming planes are determined as follows:

$$z = h_i, z_{\min} \leq h_i \leq z_{\max}$$

where $z_{\min}$ and $z_{\max}$ are minimum and maximum values of the volume Ω along the z direction. The spacing between two adjacent parallel layers is calculated by steps and imaginary plane number m.

Using the scheme above, we can obtain $(m - 2)$ cross sections and two tangent planes.

As shown in Fig. 4, domain S is supposed to be a cut-section. Assume that a series of horizontal lines $y = b_i$ ($y_{\min} \leq b_i \leq y_{\max}$) are drawn across domain S, which intersect the volume boundary B at points P_i and Q_i, and several parallel line segments P_iQ_i are obtained. According to the steps and the number of nodes, nodes on line segment P_iQ_i are also generated.

The method is used on the approximate rectangular domain. If the domain is a segmental region, the imaginary lines will be radial lines. In addition, to meet the demands of certain special domains, the imaginary planes may be composed of a series of nonparallel planes, whose equations can be defined by user.

4.3 Matching Subregions

After nodes are generated on all subregions, the subregions should be matched to form the globe domain and generate the last eight-node hexahedral mesh.

During the process of matching the subregions, there are some overlapping points on the common surface, line and point of two adjacent elements. These overlapping points must be canceled when forming the globe domain. The coordinates of boundary node will be compared with them on the adjacent subregions. If coordinates of two nodes are equal, the node must be an overlapping point. Otherwise, the node is a new point.

5. Simulation of Cylindrical Housing

Figure 5 is an aluminium alloy cylindrical housing of a helicopter. The component is forged under isothermal condition and its forming process can be divided into two stages: (a) a cylinder billet is radial extruded to form the lower part and the four ears; (b) the intermediate billet is backward extruded to form the cylindrical wall and forged the flange. In this paper, the simulation of the first forming stage is presented to show the above techniques.

Considering the symmetry, only one eighth of the whole part has been simulated. Figure 6 shows the shape of initial billet and its mesh system. The initial mesh consists of 1024 eight-node hexahedral elements and 1343 nodes.

A total of 35 time steps and four remeshing procedures are required to complete the simulation of the process. Figure 7 shows the mesh configurations before and after remeshing. The first remeshing is requested because some elements penetrate the die seriously around the die corner and the simulation accuracy becomes bad. The other remeshing procedures are performed for the mesh degeneracy.

Figure 8 shows the equivalent strain distribution at the final stage. The maximum and minimum equivalent strains are 3.26 and 0.01 at the final stage, respectively. The maximum strains occur in the inside part of the lower die cavity. The results are in agreement with them in Ref 11.

6. Conclusions

A three-dimensional FEM code for simulation of bulk forming processes has been developed. Some techniques to solve three-dimensional complicated problems are presented. The B-spline technique is adopted to describe complicated die surface. An algorithm to judge and modify the relationship between the boundary nodes and the die surface is presented. A three-dimensional remeshing technique based on the generation of eight-node hexahedral elements is proposed.

In the proposed remeshing method, the volume to be discretized is supposed to be cut by a series of parallel planes, and cross sections can be obtained. Then nodes are generated automatically layer by layer. Finally, eight-node hexahedral elements can be constructed by connecting the nodes on two adjacent cross sections. The cylindrical housing forging processes have been simulated by the system. The simulation results show that the given techniques and FEM code are reasonable and feasible.

Acknowledgments

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